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CERTAIN FINITE-DIFFERENCE SCHEMES FOR EQUATIONS OF THE NONSTATI--ETC(U)
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FOREIGN TECHNOLOGY DIVISION

CERTAIN FINITE-DIFFERENCE SCHEMES FOR EQUATIONS OF THE NONSTATIONARY LAMINAR BOUNDARY LAYER

by

A. P. Oskolkov

EDITED TRANSLATION

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CERTAIN FINITE-DIFFERENCE SCHEMES FOR EQUATIONS OF THE NONSTATIONARY LAMINAR BOUNDARY LAYER

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*<u>ye</u> initially, after vowels, and after ъ, ь; <u>e</u> elsewhere. When written as ё in Russian, transliterate as yё or ё. The use of diacritical marks is preferred, but such marks may be omitted when expediency dictates.

GREEK ALPHABET

Alpha	Α α *α*		Nu	Ν ν	
Beta	Β β		Xi	Ξ ξ	
Gamma	Γ γ		Omicron	Ο ο	
Delta	Δ δ		Pi	Π π	
Epsilon	Ε ε ϵ		Rho	Ρ ρ ϱ	
Zeta	Ζ ζ		Sigma	Σ σ ς	
Eta	Η η		Tau	Τ τ	
Theta	Θ θ ϑ		Upsilon	Υ υ	
Iota	Ι ι		Phi	Φ φ ϕ	
Kappa	Κ ϰ κ *ϰ*		Chi	Χ χ	
Lambda	Λ λ		Psi	Ψ ψ	
Mu	Μ μ		Omega	Ω ω	

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English
sin	sin
cos	cos
tg	tan
ctg	cot
sec	sec
cosec	csc
sh	sinh
ch	cosh
th	tanh
cth	coth
sch	sech
csch	csch
arc sin	$\sin^{-1}$
arc cos	$\cos^{-1}$
arc tg	$\tan^{-1}$
arc ctg	$\cot^{-1}$
arc sec	$\sec^{-1}$
arc cosec	$\csc^{-1}$
arc sh	$\sinh^{-1}$
arc ch	$\cosh^{-1}$
arc th	$\tanh^{-1}$
arc cth	$\coth^{-1}$
arc sch	sech^{-1}
arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

GRAPHICS DISCLAIMER

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CERTAIN FINITE-DIFFERENCE SCHEMES FOR
EQUATIONS OF THE NONSTATIONARY LAMINAR
BOUNDARY LAYER

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A number of stable implicit finite-difference schemes for the solution of the nonstationary Navier-Stokes ~~x~~ equations is proposed in works [1] and [2]. In this article analogies of these schemes for equations of the nonstationary laminar boundary layer are given; it is shown that these schemes in the two-dimensional and three-dimensional case are decomposed into uniform finite-difference schemes; the unique solvability of the appearing linear algebraic systems of equations is proven, and it ~~xt~~ is shown that these systems can be solved by means of a uniform trial run.

The two-dimensional nonstationary flow in the laminar boundary layer is described by the following set of equations and initial and boundary conditions [3], [4]:

$$\left.\begin{array}{c}\dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}-\nu\dfrac{\partial^2 u}{\partial y^2}=-\dfrac{\partial p}{\partial x}, \quad \dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0, \\ 0\leqslant x\leqslant X<\infty, \quad 0\leqslant y<\infty, \quad 0\leqslant t\leqslant T,\end{array}\right\} \tag{I}$$

$$u\Big|_{t=0}=\varphi(x,y), \tag{2}$$

$$u\Big|_{y=0}=0, \quad v\Big|_{y=0}=V(x,t), \quad u\Big|_{x=0}=\Psi(y,t), \tag{3}$$

$$u(x,y,t)\to U(x,t), \quad y\to\infty, \tag{4}$$

where in conformity with the Bernoulli equation

$$-\frac{\partial p}{\partial x} = U_t + UU_x \equiv F(x,t) \, . \qquad (5)$$

Furthermore, from the continuity equation, the initial condition for u and the boundary condition for v when y = 0, determined is the initial condition for v

$$v\Big|_{t=0} = V(x,0) - \int_0^y \frac{\partial \varphi}{\partial x}(x,\xi)\, d\xi \equiv \Phi(x,y) \, . \qquad (6)$$

The solution by the method of finite differences to the problem formulated by dependences (1)-(6) is virtually impossible due to the fact that the variable y is changed on the semi-infinite interval $[0,\infty]$. In work [5] the author goes around this difficulty in that by means of the Krokko [Trans. note: spelling not verified] transform $\xi = x$, $\eta = \frac{u}{U}$, $\tau = t$, $W = \frac{u_y}{U}$ transforms (1)-(6) into the initial-boundary value problem for function W, in which the variables ξ $\cdot$ η are changed in the finite limits: $0 \leq \xi \leq X$, $0 \leq \eta < 1$. On the other hand, from the theory of the boundary layer it is known [3] that $u(x,y,t)$ tends to $U(x,t)$ when $y \to \infty$ exponentially, and therefore the boundary condition (4) can be considered as being fulfilled when y = Y, where Y is a sufficiently large finite number, whereupon the admissible quantity Y can be defined according to the assigned accuracy with which the solution is sought by iterations. Therefore, subsequently with the writing out of the finite-difference scheme for the problem (1)-(6) we will assume that its solution is sought in the domain $0 \leq x \leq X$, $0 \leq y \leq Y$ and $u(x,y,t) \to U(x,t)$, $y \to Y$.

Let us divide the parallelepiped $Q_T = [0,X] \times [0,Y] \times [0,T]$ into elementary cells by planes $x_i = i\Delta x$, $y_j = j\Delta y$. $t_\ell = \ell \Delta t$ where $i = 0,1,\ldots,L$, $\Delta x = \frac{X}{L}$, $j = 0,1,\ldots,M$, $\Delta y = \frac{Y}{M}$, $\ell = 0,1,\ldots,N$, $\Delta t = \frac{T}{N}$, and we assume

$$u_{ij}^{\ell} \equiv u\left(i\Delta x, j\Delta y, \ell\Delta t\right), \quad v_{ij}^{\ell} \equiv v\left(i\Delta x, j\Delta y, \ell\Delta t\right).$$

Then by analogy with the Navier-Stokes equations [1],[2], the system of equations (1) can be approximated by the following implicit finite-difference scheme:

$$\frac{u_{ij}^{\ell+1}-u_{ij}^{\ell}}{\Delta t}+u_{ij}^{\ell}\frac{u_{ij}^{\ell+1}-u_{i-1j}^{\ell+1}}{\Delta x}+v_{ij}^{\ell}\frac{u_{ij}^{\ell+1}-u_{ij-1}^{\ell+1}}{\Delta y}-\nu\frac{u_{ij+1}^{\ell+1}-2u_{ij}^{\ell+1}+u_{ij-1}^{\ell+1}}{(\Delta y)^2}=F_{ij}^{\ell+1} \quad (7)$$

$$\frac{u_{ij}^{\ell+1}-u_{i-1j}^{\ell+1}}{\Delta x}+\frac{v_{ij}^{\ell+1}-v_{ij-1}^{\ell+1}}{\Delta y}=0, \quad (8)$$

where

$$i=1,2,\ldots,L, \quad j=1,2,\ldots,M, \quad \ell=0,1,2,\ldots, N-1.$$

Added to equations (7) and (8) are the following initial and boundary conditions:

$$u_{ij}^{0}=\varphi_{ij}, \quad v_{ij}^{0}=\Phi_{ij}, \quad i=0,1,\ldots,L, \quad j=0,1,\ldots,M. \quad (9)$$

$$\left.\begin{array}{c} u_{i0}^{\ell+1}=0, \quad v_{i0}^{\ell+1}=V_{i}^{\ell+1}, \quad u_{0j}^{\ell+1}=\Psi_{j}^{\ell+1}, \\ i=0,1,\ldots,L, \quad j=0,1,\ldots,M, \quad \ell=0,1,\ldots,N-1. \end{array}\right\} \quad (10)$$

$$u_{iM}^{\ell+1}=U_{i}^{\ell+1}\equiv U\left(i\Delta x,(\ell+1)\Delta t\right), \quad i=0,1,\ldots,L, \quad \ell=0,1,\ldots,N-1. \quad (11)$$

From equations (7) and (8) it is clear that introduced into them are values of function U on two adjacent verticals x_{i-1} and x_i, and values of function v - only on one vertical x_i, $i=1,2,\ldots,L$. By using three of the boundary conditions (10), we obtain the cyclic process for the finding of the unknowns $u_{ij}^{\ell+1}$, $v_{ij}^{\ell+1}$, on each ith step of which we have on the ith vertical at any $\ell=1,2,\ldots,N-1$ a system of 2(M-1)linear algebraic equations with 2(M-1) unknowns, and these equations should be solved by taking into account the initial and boundary conditions (9)-(11). Thereby the problem is reduced to the subsequent solution of the uniform finite-difference problems (7)-(11). Let us show that theorem 1 is correct.

The system of equations (7)-(11) is uniquely solvable at each $i=1,2,\ldots,L$ and each $\ell=0,1,\ldots,N-1$.

Since the equations (7)-(11) arethe system of linear algebraic equations in which the number of equations is equal to the number of unknowns, then for proof ~~x~~ of the theorem it is enough to show ~~akk~~ that the corresponding uniform systems have only a trivial solution. Further, starting from the uniform initial condition (9), we find that on the ith vertical when $\forall\, \ell=0,1,\ldots,N-1$ the corresponding (7), (8) uniform system has the form

$$\frac{1}{\Delta t}\, u_{ij}^{\ell+1} - \nu\, \frac{u_{i,j+1}^{\ell+1} - 2u_{ij}^{\ell+1} + u_{i,j-1}^{\ell+1}}{(\Delta y)^2} = 0, \tag{7a}$$

$$\frac{1}{\Delta x}\, u_{ij}^{\ell+1} + \frac{v_{ij}^{\ell+1} - v_{i,j-1}^{\ell+1}}{\Delta y} = 0, \tag{8a}$$

where $i=1,2,\ldots,L$, $j=1,2,\ldots,M$, $\ell=0,1,\ldots,N-1$.

Added to these equations are the uniform boundary conditions

$$u_{i0}^{\ell+1}=0\,,\ u_{iM}^{\ell+1}=0\,,\ i=1,2,\ldots,L\,,\ \ell=0,1,\ldots,N-1, \tag{12}$$

$$v_{i0}^{\ell+1}=0\,,\ i=1,2,\ldots,L\,,\ \ell=0,1,\ldots,N-1\,. \tag{13}$$

It is known [6] that the system (7a), (12) has only a trivial solution $u_{ij}^{\ell+1}=0$, and therefore from (8a) and (13) it follows that $v_{ij}^{\ell+1}=0$, $i=1,2,\ldots,L$, $j=0,1,\ldots,M$, $\ell=0,1,\ldots,N-1$. Thereby the theorem 1 is proven.

Let us point out the simple method of solving the system (7)-(11). For this let us rewrite equations (7), (8) in the following way:

$$A_{ij}^{\ell} u_{i,j-1}^{\ell+1} + B_{ij}^{\ell} u_{ij}^{\ell+1} + C_{ij}^{\ell} u_{i,j+1}^{\ell+1} = f_{ij}^{\ell} , \tag{14}$$

$$u_{ij}^{\ell+1} + \frac{\Delta x}{\Delta y} v_{ij}^{\ell+1} - \frac{\Delta x}{\Delta y} v_{i,j-1}^{\ell+1} = g_{ij}^{\ell} , \tag{15}$$

where

$$\left.\begin{aligned} A_{ij}^{\ell} &= -\left(\nu \frac{\Delta t}{(\Delta y)^2} + \frac{\Delta t}{\Delta y} v_{ij}^{\ell}\right) , \quad B_{ij}^{\ell} = 1 + \frac{\Delta t}{\Delta x} u_{ij}^{\ell} + \frac{\Delta t}{\Delta y} v_{ij}^{\ell} + 2\nu \frac{\Delta t}{(\Delta y)^2}, \\ C_{ij}^{\ell} &= -\nu \frac{\Delta t}{(\Delta y)^2} , \quad f_{ij}^{\ell} = \Delta t F_{ij}^{\ell+1} + u_{ij}^{\ell} + \frac{\Delta t}{\Delta x} u_{ij}^{\ell} u_{i-1,j}^{\ell+1} , \quad g_{ij}^{\ell} = u_{i-1,j}^{\ell+1} . \end{aligned}\right\} \tag{16}$$

Equations (14) together with the boundary conditions

$$u_{i0}^{\ell+1} = 0 , \quad u_{iM}^{\ell+1} = U_i^{\ell+1} ,$$

the initial conditions (9) and boundary condition $u_{0j}^{\ell+1} = \Psi_j^{\ell+1}$ can be solved on each ith vertical, $i = 1,2,\dots,L$ by the dispersion method [7], and after this $v_{ij}^{\ell+1}$ is determined from (15) with the use of the boundary condition

$v_{i0}^{\ell+1} = V_i^{\ell+1}$ by the formula

$$v_{ij}^{\ell+1} = v_{i,j-1}^{\ell+1} + \frac{\Delta y}{\Delta x}\left(g_{ij}^{\ell} - u_{ij}^{\ell+1}\right) , \quad j = 1,2,\dots,M .$$

The three-dimensional nonstationary flow in the laminar boundary layer, circumfluent a piece of plane (x, z), is described by the following set of equations and initial and boundary conditions [4]:

$$\left.\begin{aligned} &\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + W \frac{\partial u}{\partial z} - \nu \frac{\partial^2 u}{\partial y^2} = -\frac{\partial p}{\partial x}, \\ &\frac{\partial W}{\partial t} + u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} - \nu \frac{\partial^2 W}{\partial y^2} = -\frac{\partial p}{\partial z}, \\ &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial W}{\partial z} = 0 , \quad 0 \leq x < X , \; 0 < z \leq Z , \; 0 < y < \infty , \; 0 < t < T; \end{aligned}\right\} \tag{17}$$

$$u\Big|_{t=0}=\varphi(x,y,z)\,,\quad W\Big|_{t=0}=\Psi(x,y,z)\,; \tag{18}$$

$$u\Big|_{y=0}=0\,,\quad v\Big|_{y=0}=V(x,z,t)\,,\quad W\Big|_{y=0}=0\,; \tag{19}$$

$$\left.\begin{aligned} u\Big|_{x=0}&=\alpha(y,z,t),\quad u\Big|_{z=0}=\beta(x,y,t),\\ W\Big|_{x=0}&=\gamma(y,z,t),\quad W\Big|_{z=0}=\delta(x,y,t)\,; \end{aligned}\right\} \tag{20}$$

$$u(x,y,z,t)\to U(x,z,t)\,,\; W(x,y,z,t)\to W(x,z,t)\,,\; y\to\infty\,. \tag{21}$$

Here the derivatives of pressure $\frac{\partial p}{\partial x}\equiv F(x,z,t)$ and $\frac{\partial p}{\partial z}\equiv G(x,z,t)$ are considered to be the assigned functions. Furthermore, from the continuity equation, the initial conditions (18) for u and W and the boundary condition for v when y = 0, it is possible, just as in the two-dimensional case, to define the initial condition for v

$$v\Big|_{t=0}=V(x,z,0)-\int_0^y\left[\frac{\partial\varphi}{\partial x}(x,\xi,z)+\frac{\partial\Psi}{\partial z}(x,\xi,z)\right]d\xi=\Phi(x,y,z). \tag{22}$$

Having in mind to use the finite-difference method for solving the problem (17)-(22), according to the very same considerations as in the two-dimensional case, we replace the semi-infinite interval of the change in variable y by the finite interval [0, Y], where Y is a sufficiently large number the accessible value of which again can be found by iterations, and, thereby, we examine the problem which describes the relations (17)-(22) in the domain $0\leq x\leq X$, $0\leq y\leq Y$, $0\leq z\leq Z$, and we replace the boundary conditions (21), respectively, by the following:

$$u\to U(x,z,t)\,,\; w\to W(x,z,t)\,,\; y\to Y\,.$$

Let us divide the parallelpiped $Q_T=[0,X]\times[0,Y]\times[0,Z]\times[0,T]$ into the elementary cells by planes $x_i=i\Delta x$, $y_j=j\Delta y$, $z_k=k\Delta z$, $t_l=l\Delta t$, where $i=0,1,\dots,L$, $\Delta x=\frac{X}{L}$, $j=0,1,\dots,M$, $\Delta y=\frac{Y}{M}$, $k=0,1,\dots,R$, $\Delta z=\frac{Z}{R}$, $l=0,1,\dots,N$, $\Delta t=\frac{T}{N}$, and we assume

$$u_{ijk}^{l} \equiv u\left(i\Delta x, j\Delta y, k\Delta z, l\Delta t\right) .$$

Similarly determined are v_{ijk}^{l}, w_{ijk}^{l} and so on.

By analogy with the Navier-Stokes equations [1] and [2], the system of equations (17) can be approximated by the following implicit finite-difference scheme:

$$\frac{u_{ijk}^{l+1}-u_{ijk}^{l}}{\Delta t}+u_{ijk}^{l}\frac{u_{ijk}^{l+1}-u_{i-1,j,k}^{l+1}}{\Delta x}+v_{ijk}^{l+1}\frac{u_{ijk}^{l+1}-u_{i,j-1,k}^{l+1}}{\Delta y}+$$

$$+w_{ijk}^{l}\frac{u_{ijk}^{l+1}-u_{i,j,k-1}^{l+1}}{\Delta z}-\nu\frac{u_{i,j+1,k}^{l+1}-2u_{ijk}^{l+1}+u_{i,j-1,k}^{l+1}}{(\Delta y)^2}=-F_{ijk}^{l+1}, \tag{23}$$

$$\frac{w_{ijk}^{l+1}-w_{ijk}^{l}}{\Delta t}+u_{ijk}^{l}\frac{w_{ijk}^{l+1}-w_{i-1,j,k}^{l+1}}{\Delta x}+v_{ijk}\frac{w_{ijk}^{l+1}-w_{i,j-1,k}^{l+1}}{\Delta y}+$$

$$+w_{ijk}^{l}\frac{w_{ijk}^{l+1}-w_{i,j,k-1}^{l+1}}{\Delta z}-\nu\frac{w_{i,j+1,k}^{l+1}-2w_{ijk}^{l+1}+w_{i,j-1,k}^{l+1}}{(\Delta y)^2}=-G_{ijk}^{l+1}, \tag{24}$$

$$\frac{u_{ijk}^{l+1}-u_{i-1,j,k}^{l+1}}{\Delta x}+\frac{v_{ijk}^{l+1}-v_{i,j-1,k}^{l+1}}{\Delta y}+\frac{w_{ijk}^{l+1}-w_{i,j,k-1}^{l+1}}{\Delta z}=0, \tag{25}$$

where

$$i=1,2,\dots,L,\ j=1,2,\dots,M,\ k=1,2,\dots,R,\ l=0,1,\dots,N-1 .$$

Added to equations (23)-(25) are the following initial and boundary condtions:

$$\left.\begin{array}{l} u_{ijk}^{0}=\varphi_{ijk},\ v_{ijk}^{0}=\Phi_{ijk},\ w_{ijk}^{0}=\Psi_{ijk}, \\ i=0,1,\dots,L,\ j=0,1,\dots,M,\ k=0,1,\dots,R, \end{array}\right\} \tag{26}$$

$$\left.\begin{array}{l} u_{i0k}^{l+1}=0,\ v_{i0k}^{l+1}=V_{ik}^{l+1},\ w_{i0k}^{l+1}=0, \\ i=0,1,\dots,L,\ k=0,1,\dots,R,\ l=0,1,\dots,N-1, \end{array}\right\} \tag{27}$$

$$\left.\begin{array}{l} u_{0jk}^{l+1}=\alpha_{jk}^{l+1},\ u_{ij0}^{l+1}=\beta_{ij}^{l+1},\ i=0,1,\dots,L,\ j=0,1,\dots,M, \\ w_{0jk}^{l+1}=\gamma_{jk}^{l+1},\ w_{ij0}^{l+1}=\delta_{ij}^{l+1},\ k=0,1,\dots,R,\ l=0,1,\dots,N-1, \end{array}\right\} \tag{28}$$

$$\left.\begin{array}{l} u_{iM\kappa}^{\ell+1} = U_{i\kappa}^{\ell+1} \;, \quad w_{iM\kappa}^{\ell+1} = W_{i\kappa}^{\ell+1} \;, \\ i = 0,1,\dots,L \;, \; \kappa = 0,1,\dots,R \;, \; \ell = 0,1,\dots,N-1 . \end{array}\right\} \quad (29)$$

From equations (23)-(25) it is clear that entering into them are values of functions u, w on three adjacent verticals with numbers (i,κ), $(i-1,\kappa)$ and $(i,\kappa-1)$, and values of function v only on one vertical (i,κ). Using boundary conditions (28), we obtain the cyclic process for the finding of $u_{ij\kappa}^{\ell+1}$, $v_{ij\kappa}^{\ell+1}$, $w_{ij\kappa}^{\ell+1}$, in which on the (i,κ)-th vertical at any $\ell = 1,2,\dots,N-1$ we have a system of 3(M-1) linear algebraic equations with 3(M-1) unknowns, whereupon these equations should be solved, taking the initial and boundary conditions (26)-(29) into account. Thereby the problem (17)-(22) is reduced tothe subsequent solution of the one-dimensional finite-difference problems (23)-(29). Just as in the two-dimensional case, it x is shown that theorem 2 is correct.

The system of equations (23)-(29) is uniquely solvable at any $i=1,2,\dots,L$, $\kappa=1,2,\dots,M$, $\ell=0,1,\dots,N-1$.

The system (23)-(29) permits a simple method of solution. For proof of this, let us rewrite the equations (23)-(25) in the following way:

$$A_{ij\kappa}^{\ell} u_{i,j-1,\kappa}^{\ell+1} + B_{ij\kappa}^{\ell} u_{ij\kappa}^{\ell+1} + C_{ij\kappa}^{\ell} u_{i,j+1,\kappa}^{\ell+1} = f_{ij\kappa}^{\ell} \;, \quad (30)$$

$$A_{ij\kappa}^{\ell} w_{i,j-1,\kappa}^{\ell+1} + B_{ij\kappa}^{\ell} w_{ij\kappa}^{\ell+1} + C_{ij\kappa}^{\ell} w_{i,j+1,\kappa}^{\ell+1} = \tilde{f}_{ij\kappa}^{\ell} \;, \quad (31)$$

$$\frac{\Delta y}{\Delta x} u_{ij\kappa}^{\ell+1} + \frac{\Delta y}{\Delta z} w_{ij\kappa}^{\ell+1} + v_{ij\kappa}^{\ell+1} - v_{i,j-1,\kappa}^{\ell+1} = g_{ij\kappa}^{\ell} \;, \quad (32)$$

where

$$
\left.
\begin{aligned}
&A^{l}_{ijk}=-\left(\frac{\nu\Delta t}{(\Delta y)^2}+\frac{\Delta t}{\Delta y}v^{l}_{ijk}\right),\ B^{l}_{ijk}=1+\frac{\Delta t}{\Delta x}u^{l}_{ijk}+\frac{\Delta t}{\Delta y}v^{l}_{ijk}+\frac{\Delta t}{\Delta z}w^{l}_{ijk}+\frac{2\nu\Delta t}{(\Delta y)^2},\\
&C^{l}_{ijk}=-\frac{\nu\Delta t}{(\Delta y)^2},\ f^{l}_{ijk}=-\Delta t F^{l+1}_{ijk}+u^{l}_{ijk}+\frac{\Delta t}{\Delta x}u^{l}_{ijk}u^{l+1}_{i-1,jk}+\frac{\Delta t}{\Delta z}w^{l}_{ijk}u^{l}_{ij,k-1},\\
&\tilde{f}^{l}_{ijk}=-\Delta t G^{l+1}_{ijk}+w^{l}_{ijk}+\frac{\Delta t}{\Delta x}u^{l}_{ijk}w^{l+1}_{i-1,jk}+\frac{\Delta t}{\Delta z}w^{l}_{ijk}w^{l+1}_{i-1,jk},\\
&g^{l}_{ijk}=\frac{\Delta y}{\Delta x}u^{l+1}_{i-1,jk}+\frac{\Delta y}{\Delta z}w^{l+1}_{ij,k-1}.
\end{aligned}
\right\}\quad(33)
$$

The unknowns u^{l+1}_{ijk} and w^{l+1}_{ijk} are determined independently from each other from equations (30), (31) and the initial and boundary condtiions (26)-(27) by means of the one-dimensional dispersion,and after this the unknowns v^{l+1}_{ijk} are determined from equation (32) and the second of the boundary conditions (27) according to equation

$$
v^{l+1}_{ijk}=v^{l+1}_{ij-1,k}+g^{l}_{ijk}-\left(\frac{\Delta y}{\Delta x}u^{l+1}_{ijk}+\frac{\Delta y}{\Delta z}w^{l+1}_{ijk}\right),\ j=1,2,\ldots,M.
$$

By applying the known sufficient conditions of correctness of the one-dimensionaldispersion [7], and using the fact that according to the physical meaning of the problem $u^{l}_{ijk}\geq 0$, $w^{l}_{ijk}\geq 0$, it is easy to establish that the solutions to equations (30)-(33) are stable with respect to the computational errors if the following condition is fulfilled:

$$
\frac{\nu}{\Delta y}+\min_{ijkl} v^{l}_{ijk}>0 .
$$

In the two-dimensional case the condition of correctness of the dispersion for the system of equations (14)-(16) has the form

$$
\frac{\nu}{\Delta y}+\min_{ijl} v^{l}_{ij}>0 .
$$

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